

# The Probability of Negation of a Cruise Missile

Dr. N. S. Sivakumaran,\* Paul A. Bigelman,<sup>†</sup> Laura T. Lee,  
and Robert J. Jesionowski  
*SPARTA Inc., 1911 North Fort Myer Drive, Suite 1100, Arlington, VA 22209*

Since the ballistic missile (BM) trajectory can be determined uniquely, the probability of negation for a BM can be determined as a function of the pair of launch and target (impact) points. On the other hand, if a cruise missile (CM) is detected at some point along its route, its intended target cannot be singled out; the CM route from the launch point to its intended target point is not unique. Given the pair of launch and target points, the probability of negation for a CM is, therefore, route dependent. Here we propose a simple method to obtain the route most likely to be chosen by the enemy, given any pair of launch and target points. One may assume conservatively that the enemy would pre-plan his CM route to minimize negation along the entire route. Accordingly, among all the routes connecting a pair of launch and target points, this particular CM route is the Least Defendable Route (LDR) for the defense. Our method has three steps: (1) Define a risk function locally in terms of the probability of detection and the conditional probability of engagement and kill once detected, defined over the entire battlefield, (2) Find the LDR, which is the route of Least Cumulative Risk for the enemy, and (3) Calculate the probability of negation for this LDR. Two methods to find the LDR's are presented: Calculus of Variations and Shortest Path (Cost). A simple numerical example is given as a demonstration of our method. Finally, conclusions and suggestions for future work are made.

## I. Introduction

### Probability of negation for ballistic missiles

The *Theater Air and Missile Defense (TAMD) Capstone Requirements Document (CRD)*<sup>1</sup> requires performance by active defense to a specified level of *probability of negation*  $P_N$ . In the CRD, this is defined as

**Probability of negation ( $P_N$ ).**  $P_N$  is the probability (per target) of target destruction, deviation from intended flight path or other actions which protect the defended area from conventional, nuclear, biological or chemical effects.

(Note that in the above definition *target* refers to the enemy missile, not to the asset location). In order to evaluate the baseline TAMD architecture against the requirements posed in the CRD, it is necessary to define  $P_N$  in terms which can be used to evaluate the Family of

\*Corresponding author: [Short name: Dr. Nigel Siva]; Commercial Phone: 703-558-0036; FAX: 703-558-0045; E-mail: siva@rosslyn.sparta.com

<sup>†</sup>Deputy Director, Joint Family of Systems Interoperability, AQE Directorate, Ballistic Missile Defense Organization, 7100 Defense Pentagon, Washington, D.C. 20301-7100. Commercial Phone: 703-693-2645; FAX: 703-693-3014; E-mail: paul.bigelman@bmdo.osd.mil

Systems (*FoS*). In general, the negation of a single missile [Theater Ballistic Missile (TBM) or CM] demands success in the following three major functions:

- (I) Sensors (*SEN*),
- (II) BMC<sup>4</sup>I,<sup>‡</sup>
- (III) Weapons (*WPN*).

Each of these functions can be further divided into many components depending upon its complexity, and also there exist many inter-dependencies among them. Therefore, the precise *a priori* calculation of *FoS*  $P_N$  ( $P_{N|FoS}$  <sup>¶</sup> or simply  $P_N$ ) would require *complete information* about these inter-dependencies as well as *accurate data* of various probabilities involved in each of these components. The accuracy and performance of the  $P_N$  equation will depend, then, not on the form of the individual system's  $P_N$  equations, but rather on how well the availability and reliability of all the necessary information are modeled. One can, therefore, in

<sup>‡</sup>BMC<sup>4</sup>I is Battle Management, Command, Control, Communications, Computers and Intelligence.

<sup>¶</sup>Here  $N|FoS$  is read "Negation given any specific Family of Systems", a notation adopted from conditional probability theory albeit nothing to do with it.

1991214063

a general way define  $P_{N|FoS}$  as a function

$$P_{N|FoS} = f(P_{SEN}, P_{BMC^4I}, P_{WPN}), \quad (1)$$

where  $P_{SEN}$ ,  $P_{BMC^4I}$  and  $P_{WPN}$  denote the probabilities of success for the Sensor, the BMC<sup>4</sup>I and Weapon components, respectively.

The Family of Systems,  $P_{N|FoS}$  and the Scenario Effectiveness  $P_{SE|FoS}$  for the *Theater Ballistic Missile Defense* (TBMD) have been defined and extensively studied; see for example Sivakumaran *et al.*<sup>2</sup>. For Cruise Missile Defense (CMD), however, proper definitions for  $P_{N|FoS}$  and  $P_{SE|FoS}$  and how to calculate them have not been well established. Here we propose a method to calculate  $P_{N|FoS}$  for the CMD; then  $P_{SE|FoS}$  can follow from it for a scenario of many CM's.

### Probability of negation for cruise missiles

Since the BM trajectory can be determined uniquely once the launch point and the burn-out conditions have become available (which in turn predict the impact point uniquely),  $P_{N|FoS}$  for a BM can be calculated simply as a function of the threat (launch) point  $T$  and the intended asset (impact or target) point  $A$ :

$$P_{N|FoS}^{BM} = P_{N|FoS}^{BM}(T, A). \quad (2)$$

On the other hand, the main source of difficulty in the CMD is that the CM route, from a given threat location  $T$  to an asset location  $A$ , cannot be predicted uniquely because it is programmed by the enemy, depending upon his perception of the defense, so that his CM has the minimum *risk* of being negated while in transit. In general, for CM's

$$P_{N|FoS}^{CM} = P_{N|FoS}^{CM}(T, A, CM\ route). \quad (3)$$

Consequently, a CMD analysis which is similar to that of TBMD necessitates *standardizing the route* for all CM's, which is discussed below.

Consider the following two important concepts:

- C1.** A good defense knows its own weakness; i.e., the degree of defense deficiency (or the extent of protection) in every area under its guard.
- C2.** The enemy wants the highest probability of success in destroying the defense's assets.

Accordingly, in order to gauge the performance of a particular defense architecture and to justify the choice of its specific beddown in the theater, the defense must consider the worst enemy attack that could occur. The minimum defense-evaluation, therefore, can be achieved

by pretending that the enemy possesses the complete information about the defense's weakness.

Since we assume that the enemy has the knowledge of the defense's weakness, in order to maximize his chance of successfully destroying an asset, he will attempt to plan his CM route through the weakest possible points in the defense. In a theater, detection may be possible at some points where a successful engagement cannot be supported, while other points may permit no detection at all; the enemy will prefer these as waypoints for his CM. Some kind of *local parameter associated with negation* (i.e., in terms of local detection, engagement and kill), therefore, is appropriate for constructing a risk function  $R$ , in terms of *risk per unit route length*, in the domain containing both the threat locations and the defended area. Then, we assume that the enemy's choice for his CM route is the one that minimizes the cumulative risk  $J$ ,

$$J \equiv \int_{s_T}^{s_A} R ds, \quad (4)$$

along the entire route from his threat (launch) point  $T$  to the intended asset (target) point  $A$ , where  $ds$  is an infinitesimal arc length along the route;  $s_T$  and  $s_A$  denote arc lengths along the route corresponding to points  $T$  and  $A$ , respectively. This particular route is labeled as the ***Least Defendable Route*** (LDR) for the defense, which corresponds to the route of Least Cumulative Risk for the enemy. Thus the CM route is customized for each pair of threat and asset points and the route dependence in (3) is eliminated. We call this "*standardizing the CM route*".

An appropriate calculation of  $P_{N|FoS}$  for CM's must therefore invoke the following three steps:

- S1.** Define a suitable ***Risk function***  $R$  in a domain which contains both the threat locations and the defended area.
- S2.** Find the ***LDR*** from a threat location to an asset location; i.e., the route of Least Cumulative Risk (exposure) for the enemy.
- S3.** Calculate the  $P_{N|FoS}$  for that LDR.

## II. Cruise missile route

### The Least Defendable Route

Let the asset region  $\mathcal{A}$  with boundary  $\partial\mathcal{A}$  be defined in the  $xy$ -plane. For simplicity, consider a single designated asset located at  $A \equiv (x_A, y_A) \in \mathcal{A}$  and a single threat located at  $T \equiv (x_T, y_T)$ . One could assume that the enemy will pick a route  $y = y^*(x)$  to fly his CM from  $T$  to  $A$  which offers the *least cumulative risk*. Consequently, among all the routes  $y = y(x)$  from  $T$  to  $A$ , this

particular route  $y=y^*(x)$  of least cumulative risk poses the greatest danger to the asset  $A$  and therefore must have the minimum  $P_{N|FoS}$  for this  $(T, A)$ -pair.

The LDR depends upon the enemy's perception of the weakness in the defense. For the time being, without loss of generality, let us assume that the enemy defines a **risk function**, in terms of risk per path length,

$$R(x, y, y') > 0 \quad (5)$$

in the  $xy$ -domain  $\mathcal{A}$ , where the prime in  $y'$  refers to the derivative with respect to the independent variable  $x$  (i.e., the operation  $d/dx$ ). This includes the cases where the risk depends strongly upon the direction  $y'$  of the CM as well as its position  $(x, y)$ . The best candidate for  $R(x, y, y')$  will be selected later in Section III.

Let  $s$  denote the **arc length** along a CM path  $y=y(x)$  from  $T$  to  $A$ ; then the **cumulative risk**  $J[y(x)]$  associated with this path  $y=y(x)$  from  $T$  to  $A$  can be written by

$$J[y(x)] \equiv \int_{s_T}^{s_A} R(x, y(x), y'(x)) ds. \quad (6)$$

The problem now is to find the CM route  $y=y^*(x)$  which minimizes the cumulative risk given by (6).

## Methods to find the LDR

### (a) Calculus of Variations

Equation (6) can be rewritten as

$$J[y(x)] = \int_{x_T}^{x_A} F(x, y, y') dx, \quad (7)$$

where

$$F(x, y, y') \equiv R(x, y, y') \sqrt{1 + y'^2}. \quad (8)$$

The enemy's aim is to find the CM route  $y=y^*(x)$  which minimizes the cumulative risk given by (7). This type of problem of finding a function  $y$  to extremize an integral is dealt with in the *Calculus of Variations* where  $y=y(x)$  is given by the solution of the **Euler-Lagrange equation**:<sup>3</sup>

$$\frac{d}{dx} \left( \frac{\partial F}{\partial y'} \right) - \frac{\partial F}{\partial y} = 0. \quad (9)$$

Substituting (8) into (9) gives the Euler-Lagrange equation for  $y(x)$  explicitly in terms of the function  $F(x, y, y')$  as<sup>3</sup>

$$F_{y'y'} \frac{d^2 y}{dx^2} + F_{y'y} \frac{dy}{dx} + (F_{y'x} - F_y) = 0 \quad (10)$$

where subscripts denote partial derivatives with respect to the variables; e.g.,  $F_{y'y'} \equiv \partial^2 F / \partial y'^2$ . The second-order ordinary differential equation (10) require two boundary conditions for its unique solution; these are given by the locations of the end points  $T$  and  $A$  of the LDR; viz.,

$$y(x_T) = y_T, \quad (11)$$

$$y(x_A) = y_A, \quad (12)$$

One can solve (10) subject to boundary conditions (11) and (12) for the LDR using numerical integration algorithms.

The CM route obtained using the Calculus of Variations has an analogy in geometrical optics. Consider the **Fermat Principle** in geometrical optics<sup>4</sup>. In 1657, Pierre de Fermat stated: *If light travels from point  $P_1$  to point  $P_2$  through any optical system, it will follow a path such that the time of travel is stationary (minimum) with respect to neighboring, but not physically possible, paths.*

Consider an optically inhomogeneous and anisotropic medium with **refractive index**  $n(x, y, y')$ . Then, the speed of light  $v$  in this medium varies according to

$$v(x, y, y') = \frac{c}{n(x, y, y')}, \quad (13)$$

where  $c$  denotes the speed of light *in vacuo*. The Fermat principle states that the actual path of light  $y=y^*(x)$  from point  $P_1 \equiv (x_1, y_1)$  to point  $P_2 \equiv (x_2, y_2)$  in this medium is such that the travel (transit) time given by

$$J[y] \equiv \int_{s_1}^{s_2} \frac{ds}{v} = \frac{1}{c} \int_{s_1}^{s_2} n(x, y, y') ds \quad (14)$$

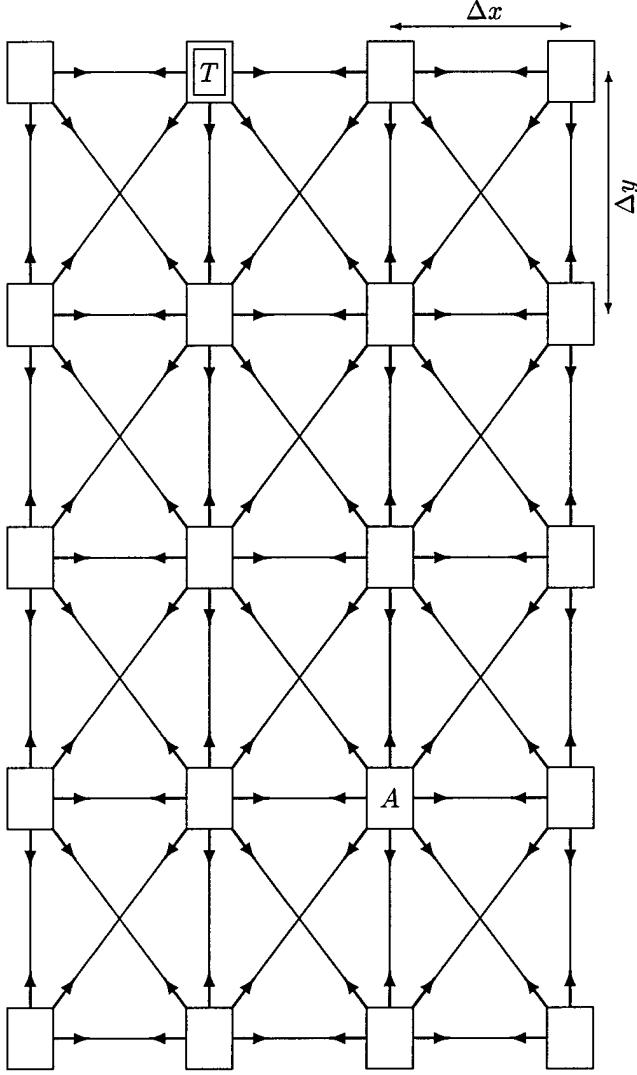
is the minimum. Here  $s_1$  and  $s_2$  are arc lengths along the light path  $y=y^*(x)$  corresponding to points  $P_1$  and  $P_2$ , respectively.

Comparing (6) with (14), the problem of finding the CM route that has the least cumulative risk with respect to the risk function  $R(x, y, y')$  is equivalent to applying the Fermat principle in geometrical optics with respect to the refractive index  $n(x, y, y')$ :

$$\underbrace{R(x, y, y')}_{\begin{array}{l} \text{Risk function in} \\ \text{CM problem} \end{array}} \iff \underbrace{n(x, y, y')}_{\begin{array}{l} \text{Refractive index in} \\ \text{geometrical optics} \end{array}}$$

The CM problem is therefore called a **Fermat-type problem**.

When the problem does *not* depend upon the CM direction  $y'$ , then the Calculus of Variations approach is altogether feasible and practicable to solve. When  $y'$  is also included, however, the calculations become very lengthy. We have therefore not adopted this cumbersome method because of the lack of sufficient time.



**Figure 1: Example of a battlefield.** The risk per unit length is given along the eight unit vectors at every node. Node  $T$ , for example, is assumed to be the threat location and all other nodes are considered as asset locations  $A$ 's. The CM can travel from one node to another along the connections (links) shown. Every node is a potential waypoint for the CM.

#### (b) Shortest-Path (Cost) Method in Operations Research

Since our problems generally include  $y'$  dependence, we will therefore go to a simple and shorter method to find the LDR, to be described below. Consider a battlefield which includes the defended area and the threat locations. Let the risk be given as *discrete* values on a regular grid of resolution  $\Delta x$  and  $\Delta y$ ; i.e.,  $R$  is given at every grid point which is hereafter referred to as a **node**. For simplicity, the threat locations and the asset locations are assumed to be at some of these

nodes. Every interior node is connected to its eight immediate neighboring nodes while a node at a corner is connected to its three immediate neighbors and a node at a boundary is connected to its immediate five. Figure 1 depicts an example of a typical network of nodes and the connections between them, for simplicity in a rectangular domain. Let there be a total of  $N$  number of nodes. From the threat node  $T$  the CM is launched at asset node  $A$ . The enemy's aim is to choose the subset  $\mathcal{W}$  which is a *node sequence* from  $N$  as the set of *waypoints* for his CM, where the first element of  $\mathcal{W}$  is  $T$  and the last element is  $A$ , so that the resulting CM route has the least cumulative risk  $J$  which is now redefined discretely from (6) as

$$J[\mathcal{W}] \equiv \sum_{\substack{j=1 \\ j \in \mathcal{W}}}^{j=A} R_j \Delta s_j, \quad (15)$$

where  $\Delta s_j$  is the distance between nodes, which is either  $\Delta x$ ,  $\Delta y$  or  $\sqrt{(\Delta x)^2 + (\Delta y)^2}$ .

The problem of minimizing the  $J$  defined by (15) is dealt with in the field of Operations Research where it is a classical optimization problem called "Finding the Shortest Path (Cost)". Various methods are discussed in Rardin<sup>6</sup> and Bertsekas<sup>7</sup> to find the shortest path from one node to all other nodes when the risk  $R$  is nonnegative, such as the Bellman–Ford Algorithm, Dijkstra Algorithm, etc. Gallo *et al.*<sup>8</sup> give a bibliography on the "Shortest Paths". Here we use the D'Esopo–Papa Algorithm developed by Gallo and Pallottino<sup>9</sup> to find the LDR's for the CM's. A computer code of this algorithm (in Fortran) is available in the book by Bertsekas.<sup>7</sup>

### III. Choice of the risk function Risk derived from the Poisson distribution

#### (a) The Poisson distribution

Cramér<sup>10</sup> and Parzen<sup>11</sup> give general derivations and applications of the Poisson distribution. Here we specialize the Poisson distribution to the CM problem. Define

D1.  $\mathcal{E} \equiv$  The event of *negating* a CM,

D2.  $X_s \equiv$  The number of CM *negations* within an interval  $[s_0, s_0 + s]$  along a specific route, counted for very many CM flights (i.e., the occurrences of event  $\mathcal{E}$ ), where  $s$  is measured from an arbitrary fixed point  $s_0$  in the route ( $X_s \geq 0$  and it is non-decreasing with  $s$ ),

D3.  $k(x, y, y') (\geq 0) \equiv$  The average number of *negations* (events  $\mathcal{E}$ ) along a given unit tangent vector (i.e., within a directed line segment of unit

length) in a route with direction  $y'$ , averaged over a large number of CM flights in this interval of the route,

**D4.**  $P[X_s = m] \equiv$  The **probability** of exactly  $m$  number of negations within an interval  $[s_0, s_0 + s]$  in the route described in **D3** for one CM flight.

We are here stating that the random distribution of the events  $\mathcal{E}$  over a long route which consists of many of the above CM flights strung together, will conform to the Poisson distribution.<sup>10,11</sup> Then the probability of exactly  $m$  number of negations within an interval  $[s_0, s_0 + s]$  is given by

$$P[X_s = m] = \frac{(ks)^m}{m!} e^{-ks}, \quad (16)$$

where the integer  $m \geq 0$  and  $k$  is the constant average number in **D3**. Expression (16) is known as the *Poisson distribution* with parameter  $k$ .

### (b) Derivation of the risk function

From (16), the probability of *no negation* (i.e., escape or leakage) of a CM along its entire route, from launch  $s=s_T$  to target  $s=s_A$ , is given by

$$P[X_s = 0] = \exp(-ks), \quad (17)$$

where

$$s \equiv s_A - s_T \quad (18)$$

if the average number of negations  $k$  per unit arc length along the CM route is assumed to be a constant. On the other hand, if this average number varies along the CM route  $y=y(x)$ , then one can generalize (17) to

$$P[X_s = 0] = \exp(-J), \quad (19)$$

where

$$J \equiv \int_{s_T}^{s_A} k(x, y, y') ds. \quad (20)$$

Consequently, one may assume that the enemy would choose that path  $y=y^*(x)$  for his CM which **maximizes**  $P[X_s = 0]$ . This is equivalent to finding the CM route  $y=y^*(x)$  (i.e., the LDR) which **minimizes**  $J$  given by (20). Comparing (20) with (6), one therefore defines the risk function as

$$R \equiv k(x, y, y'). \quad (21)$$

The probability of negation of the CM taking the LDR is then directly given by

$$\begin{aligned} P_N|_{\min} &= 1 - P[X_s = 0]|_{\max} \\ &= 1 - \exp(-J_{\min}). \end{aligned} \quad (22)$$

### The parameter $k$ in terms of probabilities

We define

**P1.**  $p_d(x, y, y')$   $\equiv$  The probability of detection of a CM within a unit vector in the route for one CM flight,

**P2.**  $p_{ek}(x, y, y')$   $\equiv$  Given detection, the *conditional* probability of engagement and kill of the CM within the same unit vector.

In definition **D2**,  $X_s$  is the number of CM negations within an interval  $[s_0, s_0 + s]$ , where  $s$  is measured from an arbitrary fixed point  $s_0$  in the route. Letting this arbitrary point  $s_0 \equiv s$  and considering a unit interval  $[s, s+1]$  with direction  $y'$ , define  $X$  as the number of CM negations along this unit vector. This number  $X$  will generally be a fraction much less than one. Then the *local probability of no negation*  $q_n$  of a CM along this unit vector can be written as

$$P[X = 0] = \exp(-k) \quad (23)$$

$$= 1 - p_d p_{ek} \equiv q_n, \quad (24)$$

where (23) follows from (16) by setting  $m=0$  and  $s=1$  while (24) follows from **P1** and **P2**. From (21) and taking the natural logarithm of (23) and (24), one gets the risk function  $R$  in terms of the *local probability of no negation*  $q_n$ :

$$R = k(x, y, y') = -\ln q_n(x, y, y'). \quad (25)$$

Note that if  $Q_n$  is the local probability of no negation which is calculated in a line segment of length  $L$ , then the local probability of no negation  $q_n$  in a unit length is given by  $q_n = Q_n^{1/L}$ .

### IV. Example

Consider a rectangular battlefield of size 9 km in the  $x$ -direction and 16 km in the  $y$ -direction. On a regular grid of resolution  $\Delta x = 3$  km and  $\Delta y = 4$  km, the product  $p_d \times p_{ek}$  is shown along eight unit vectors (which cover the range for  $y'$ ) at every node as depicted in Figure 2, and the risk  $R$  is given by (25); viz.,

$$R = -\ln q_n = -\ln(1 - p_d p_{ek}). \quad (26)$$

For simplicity the risk (per unit length) is assumed constant between adjacent nodes. In order somewhat to smooth down the data in this discrete model, we take this constant as the average of the risk values at adjacent nodes for the same direction, e.g., going southeast from node 5 [ $R = -\ln(1 - .10) = 0.1054$ ] to node 10 [ $R = -\ln(1 - .60) = 0.9163$ ] we take the average 0.5108 as the risk per unit length. Node number 2 is taken to be the threat location ( $T$ ) while the rest of the nodes are considered as asset locations. The LDR's from node 2 to every other node are shown in Figure 3 and the minimum probability of negation  $P_N|_{\min}$  given by (22) is also marked next to every node.

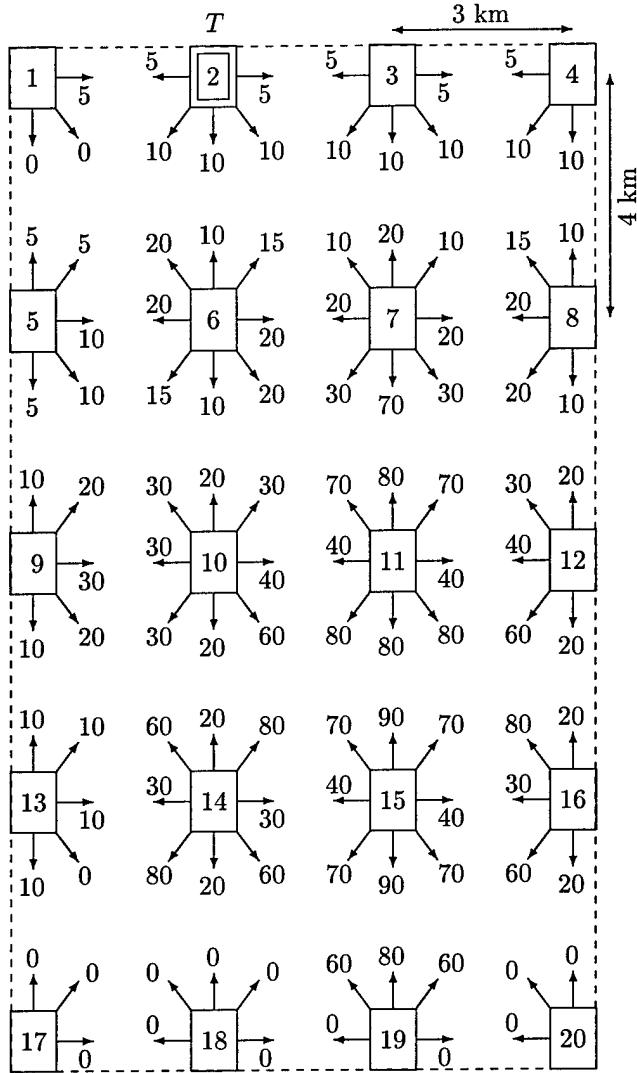


Figure 2: Example of a battlefield. The product  $p_d \times p_{ek}$  along the eight unit vectors at every node is given as percentage. Node number 2 is assumed to be the threat location  $T$  and all other nodes are considered as asset locations  $A$ 's.

## V. Conclusions

Finding the probability of negation  $P_N$  for CM's is a difficult problem, for the CM route is determined by the enemy. Here we have presented a simple method to calculate  $P_N$  by considering the enemy's most favorite CM route—a concept we called “standardizing the CM route”. Detection alone is insufficient to define a reasonable risk function because, for a successful negation, it has to be supported by engagement and there may be no engagement units within proximity. Any reasonable risk function must contain the three elements: detection, engagement and kill. The important step is, therefore, the construction of an appropriate risk function. Our methodology is demonstrated by taking local

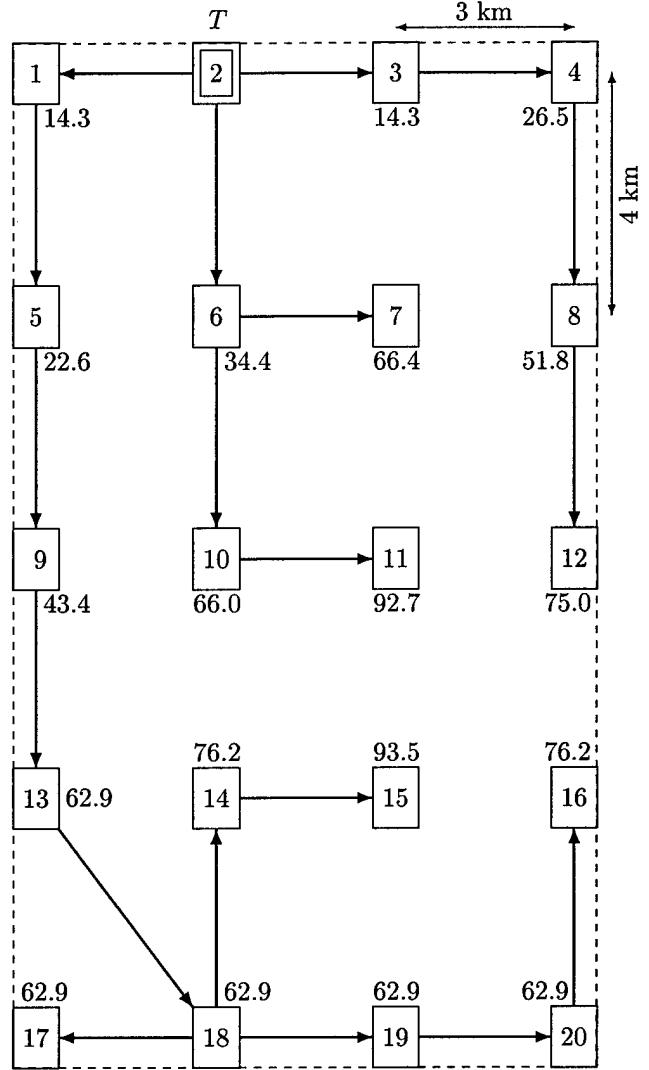


Figure 3: This shows the Least Defendable Route to every asset node from the threat location  $T$  (node number 2). The number marked adjacent to every node is the minimum probability of negation  $P_N|_{\min}$  (given as percentage) for the LDR from  $T$  to that node. This figure, therefore, can be interpreted as the minimum  $P_N$  contour map corresponding to this particular threat  $T$ .

probability of negation, which is defined as the product of the local probability of detection along a unit vector and the conditional probability of engagement and kill given that detection, to define a risk function and then finding the Least Defendable Route the enemy would prefer by minimizing the negation probability taken over the entire route. A Shortest-Path (Cost) Algorithm from Operations Research has been used to determine the LDR's from a single threat to many assets and the minimum  $P_N$  contour map resulting from a single threat has been constructed.

There is another important problem which can be directly solved using our method: Given a fixed asset location to find the threat location which provides the LDR when the enemy has a choice of various threat locations.

It is known that CM's have aerodynamic constraints and the number of turns they can make is limited. The algorithm we used excludes these restrictions. An algorithm minimizing (15) subject to constraints such as the magnitude of turns and the number of turns would justify further study.

*ory and some of its applications*, Robert E. Krieger Publishing Company, Huntington, N. Y., Reprint 1973, pp. 102–108 [Equation (6.5.2) on p. 105].

<sup>11</sup>Parzen, Emanuel, *Modern Probability Theory and its Applications*, John Wiley and Sons, Inc., N. Y., 1992, pp. 251–267 [Equation (5.17) on p. 267].

### Acknowledgments

The helpful suggestions given by Mr. John L. Dyer and Dr. Sean K. Collins are highly appreciated. The first author also wants to thank Prof. Dr. Robert F. Dressler (Mathematician) for many discussions and suggesting the Poisson distribution for the problem.

### References

<sup>1</sup>United States Atlantic Command, *Theater Missile Defense Capstone Requirements Document*, Classified by: Multiple Sources, DECL: OADR, Second Draft, 03 December 1997, p. GL-10.

<sup>2</sup>Sivakumaran, N.S., Vicki Huo and Laura T. Lee, "Mathematical Analysis of Probabilities of Negation," *Technical Report*, No. ASTO/TR: 98-001, (SPARTA, Inc., 1911 North Fort Myer Drive, Suite 1100, Arlington, VA 22209), 84pp.

<sup>3</sup>Hildebrand, Francis B., *Methods of Applied Mathematics*, Second Edition, Dover Publications, Inc., N.Y., 1992, p. 125 [Equations (17a) and (17b)].

<sup>4</sup>Welford, W.T.: (1988), *Optics*, Third Edition, Oxford University Press, pp. 19–20

<sup>5</sup>Edward L. O'Neill, (1992): *Introduction to Statistical Optics*, Dover Publications, Inc, N.Y., Section 3–1, pp. 30–32.

<sup>6</sup>Rardin, Ronald L., *Optimization in Operations Research*, Prentice Hall, N. J., 1998, pp. 409–474.

<sup>7</sup>Bertsekas, Dimitri P., *Linear Network Optimization: Algorithms and Codes*, The MIT Press, Cambridge, Massachusetts, 1992, pp. 60–89; pp. 262–266.

<sup>8</sup>Gallo, G. S., Pallottino, S., Ruggen, C., and Starchi, G., "Shortest Paths: A Bibliography," Sofmat Document 81-P1-4-SOFMAT-27, Rome, Italy, 1982.

<sup>9</sup>Gallo, G. S., Pallottino, "Shortest Path Algorithms", *Annals of Operations Research*, Vol. 7, 1988.

<sup>10</sup>Cramér, Harald, *The Elements of probability The-*

## AIAA 1998 Missile Sciences Conference

Title of Document

The Probability of Negation of a Cruise Missile  
 By N. S. Sivakumaran et al.

## PLEASE CHECK THE APPROPRIATE BLOCK BELOW:

 3

copies are being forwarded. Indicate whether Statement A, B, C, D, E, F, or X applies.

- DISTRIBUTION STATEMENT A:  
APPROVED FOR PUBLIC RELEASE: DISTRIBUTION IS UNLIMITED
- DISTRIBUTION STATEMENT B:  
DISTRIBUTION AUTHORIZED TO U.S. GOVERNMENT AGENCIES ONLY: (Indicate Reason and Date). OTHER REQUESTS FOR THIS DOCUMENT SHALL BE REFERRED TO (Indicate Controlling DoD Office).
- DISTRIBUTION STATEMENT C:  
DISTRIBUTION AUTHORIZED TO U.S. GOVERNMENT AGENCIES AND THEIR CONTRACTORS: (Indicate Reason and Date). OTHER REQUESTS FOR THIS DOCUMENT SHALL BE REFERRED TO (Indicate Controlling DoD Office).
- DISTRIBUTION STATEMENT D:  
DISTRIBUTION AUTHORIZED TO DoD AND U.S. DoD CONTRACTORS ONLY: (Indicate Reason and Date). OTHER REQUESTS SHALL BE REFERRED TO (Indicate Controlling DoD Office).
- DISTRIBUTION STATEMENT E:  
DISTRIBUTION AUTHORIZED TO DoD COMPONENTS ONLY: (Indicate Reason and Date). OTHER REQUESTS SHALL BE REFERRED TO (Indicate Controlling DoD Office).
- DISTRIBUTION STATEMENT F:  
FURTHER DISSEMINATION ONLY AS DIRECTED BY (Indicate Controlling DoD Office and Date) or HIGHER DoD AUTHORITY.
- DISTRIBUTION STATEMENT X:  
DISTRIBUTION AUTHORIZED TO U.S. GOVERNMENT AGENCIES AND PRIVATE INDIVIDUALS OR ENTERPRISES ELIGIBLE TO OBTAIN EXPORT-CONTROLLED TECHNICAL DATA IN ACCORDANCE WITH DoD DIRECTIVE 5230.25. WITHHOLDING OF UNCLASSIFIED TECHNICAL DATA FROM PUBLIC DISCLOSURE, 6 Nov 1984 (Indicate date of determination). CONTROLLING DoD OFFICE IS (Indicate Controlling DoD Office).
- This document was previously forwarded to DTIC on \_\_\_\_\_ (date) and the AD number is \_\_\_\_\_
- In accordance with provisions of DoD instructions, the document requested is not supplied because:
- It will be published at a later date. (Enter approximate date, if known).
- Other. (Give Reason) The Public Release Statement from Pentagon is underway and will be provided to you soon.

DoD Directive 5230.24, "Distribution Statements on Technical Documents," 18 Mar 87, contains seven distribution statements, as described briefly above. Technical Documents must be assigned distribution statements.

  
N. S. Sivakumaran  
 Authorized Signature/Date

N. S. Sivakumaran  
 Print or Type Name  
703-558-0036 (Ext. 292)  
 Telephone Number